Simulation of Soil-Structure Interaction Effects by Discrete-Time Recursive Filters

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Foundation impedance functions provide a simple means to account for soil-structure interaction (SSI) effects in dynamic response analysis of structures under seismic loads. However, the frequency-dependency of impedance functions makes it difficult to incorporate SSI in standard time-history analysis routines. This paper presents a method to transform frequency-domain impedance functions into time-domain recursive filters. The method is based on the least-squares approximation of impedance functions by ratios of two complex polynomials. Such ratios are equivalent, in the time-domain, to discrete-time recursive filters, which are simple finite-difference equations representing the relationship between foundation forces and displacements. Recursive-filter representation of impedance functions makes it very easy to incorporate soil-structure interaction in standard time-history analysis.

INTRODUCTION

It is well known that soil-structure interaction (SSI) is one of the critical factors influencing response and damage in structures during earthquakes. The primary effects of SSI are that it lowers the dominant frequency of the structure's vibrations, particularly for heavy structures founded on soft soils, filters high frequencies, and increases damping (Safak, 1995). Depending on the frequency content of the ground shaking, SSI can be detrimental or beneficial for the structure. SSI becomes detrimental if, because of SSI, the dominant frequency of vibrations becomes closer to the dominant frequency of ground shaking. For structures susceptible to SSI, it is important that the frequency with SSI is used in calculating seismic forces and displacements because they are both proportional to the square of natural frequency.

A simple way to incorporate SSI in seismic analysis is to model the flexibility of the soil around the foundation by using springs and dashpots. The characteristics of the springs and dashpots are defined by foundation impedance functions. The impedance function is the ratio of a harmonic force applied to the foundation to the resulting harmonic displacement at the bottom of the foundation. Impedance functions are

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functions of frequency, as well as the foundation geometry and the characteristics of soil media. The fact that impedance functions are frequency dependent makes is difficult to incorporate SSI in routine dynamic analysis. Standard time-history analysis packages cannot model frequency-dependent springs and dashpots.

This paper presents a simple method to incorporate SSI in standard time-history analysis. The method is based on the concept of matching impedance functions by a ratio of two complex polynomials. Such ratios correspond to transfer functions of discrete-time recursive filters, which are time-domain finite-difference equations representing the relationship between foundation forces and displacements.

DISCRETE-TIME RECURSIVE FILTERS FOR IMPEDANCE FUNCTIONS

A discrete-time recursive filter is defined by the following equation:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_m y(t-m) + b_0 x(t) + b_1 x(t-1) + b_2 x(t-2) + \cdots + b_n x(t-n) \quad (1)$$

where *x(t)* and *y(t)* are the original (i.e., input) and the filtered (i.e., output) signals, respectively, and a_j and b_j denote the filter coefficients. The parameter *t* is used to denote time, as well as the time index (i.e., $t \equiv t \cdot \Delta$, where Δ is the sampling interval). If the filter parameters a_j and b_j are constants the filter is a time-invariant filter, if they change with time the filter is a time-varying filter. Time-varying filters can be used to represent nonlinear systems. By taking the Fourier transform of Eq. 1, we can write the following equation for the transfer function, $H(\omega)$, of the filter

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m}} \quad \text{where } z = e^{i\omega\Delta} \text{ and } i = \sqrt{-1} \quad (2)$$

$X(\omega)$ and $Y(\omega)$ denote the complex Fourier transforms of *x(t)* and *y(t)*.

The general form of foundation impedance functions, $K(\omega)$, is

$$K(\omega) = K_0 \cdot \left[K_1(\omega) + i \cdot K_2(\omega) \right] \quad (3)$$

where K_0 denotes the static stiffness. If the foundation moves by an amount *u(t)* relative to the surrounding soil, the force, $F(t,\omega)$, exerted on the foundation by the soil is

$$F(t,\omega) = K(\omega) \cdot u(t) \quad (4)$$

$K(\omega)$ can be considered as a filter that converts *u(t)* into $F(t,\omega)$. If we can approximate $K(\omega)$ as a ratio of two complex polynomials, similar to $H(\omega)$ as shown in Eq. 2, we can then write the relationship between u and F by a discrete-time recursive filter in a form similar to that given by Eq. 1.

We can find discrete-time transfer functions in the form of Eq. 2 to match given impedance functions by using the least-squares approximation technique. We determine the parameters a_j and b_j of the discrete-time transfer function $H(\omega)$ such that it is as close to $K(\omega)$ as possible. This is accomplished by minimizing the following error function:

$$V = \sum_{\omega} W(\omega) \cdot \left[H(\omega) - K(\omega) \right]^2 \tag{5}$$

where $W(\omega)$ is the weighting function. The use of weighting function gives the flexibility of having better match between $H(\omega)$ and $K(\omega)$ at specified frequency bands. The filter parameters are determined by making

$$\frac{\partial V}{\partial a_k} = 0 \text{ and } \frac{\partial V}{\partial b_l} = 0 \text{ for } k = 1, \cdots, m \text{ and } l = 1, \cdots, n \tag{6}$$

The resulting equations for a_k and b_l are solved by using various algorithms that are available in the literature (e.g., Levy, 1959; Sanathanan and Koerner, 1962). Once the parameters of the discrete-time filter for $K(\omega)$ is determined, the spring force $F(t)$, at time step t, simply becomes

$$\begin{aligned} F(t) = -a_1 F(t-1) - a_2 F(t-2) - \cdots - a_m F(t-m) \\ + b_0 u(t) + b_1 u(t-1) + b_2 u(t-2) + \cdots + b_n u(t-n) \end{aligned} \tag{7}$$

Note that this expression is completely in the time domain. The key point for applications is that, in order to calculate $F(t)$, we need to save the past m values of $F(t)$ and past n values of $u(t)$ at every time step. This requires a simple modification in standard time-history analysis routines.

One practical problem is the selection of filter orders, m and n. There are no clear-cut rules for this selection. The higher the filter order (particularly the m value) the better the match. However, in order to have a stable filter, the poles of the filter (i.e., the roots of the denominator polynomial) should all be inside the unit circle in the complex plane. Too high m values my result in unstable filters. More detail on these and other practical points for applications are given in Safak (2004).

EXAMPLE

As an example, consider the horizontal impedance function of a circular foundation on the surface of a homogenous soil media (uniform half-space) as shown in Figure 1. The properties of the foundation and soil media are given in the figure.

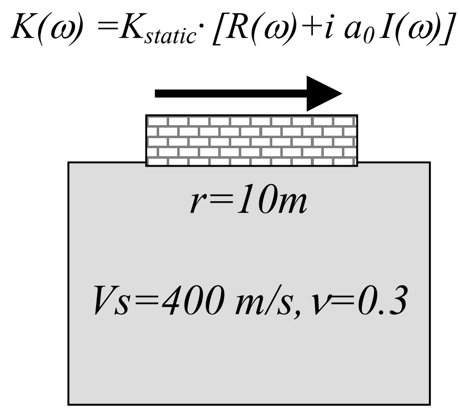

Figure 1 – Properties of the foundation and soil used in the example.

The horizontal impedance function of this foundation is given by the following equation (Veletsos and Wei, 1971):

$$K(\omega) \;= K_{static}\cdot\,[R(\omega)+i\cdot a_0\cdot I(\omega)] \quad \text{with} \quad a_0=\frac{\omega\cdot r}{V_s} \quad \text{and} \quad K_{static}=\frac{8Gr}{2-\nu} \qquad (8)$$

where K_{static} is the static stiffness; G, V_s, and ν are respectively the shear modulus, shear wave velocity, and Poisson's ratio of the soil; and r denotes the radius of the circular foundation. For the numerical values given in Figure 1, the variations of the parameters $R(\omega)$ and $I(\omega)$ of the stiffness with the non-dimensional frequency $a_0 = \omega\, r/\, V_s$ are plotted in Figure 2.

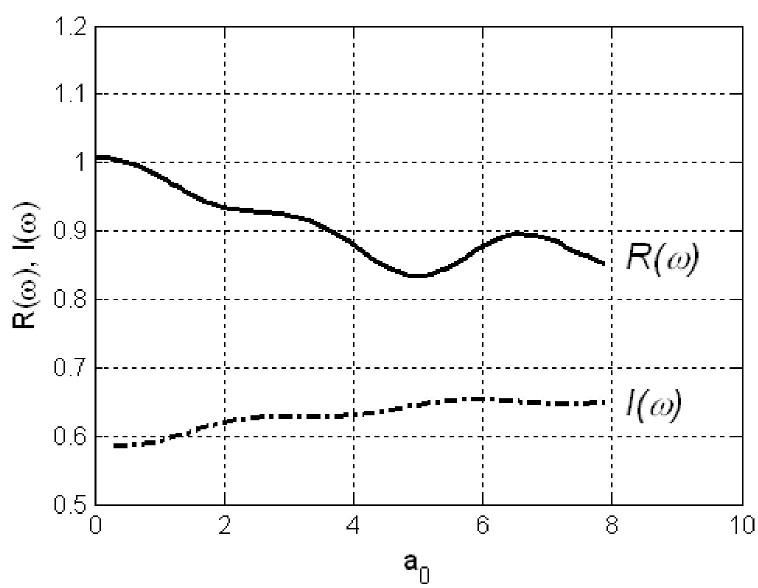

Figure 2 – Components of the horizontal impedance function of a circular foundation.

We determine the coefficients of a matching discrete-time recursive filter by assuming $m=2$, $n=1$, and using the least-squares criterion and a linearly-decaying weighting function (in order to get a better match at lower frequencies). The filter coefficients are: $a_1=1.1599$, $a_2=1.1599$, $b_0=4.3033$, and $b_1=-1.9780$. Therefore, the discrete-time transfer function for the impedance function is

$$H(\omega) = K_{static} \cdot \frac{4.3033 - 1.9780z^{-1}}{1 + 1.1599z^{-1} + 1.1599z^{-2}} \tag{9}$$

The comparison of the amplitude and phase spectrum of $H(\omega)$ with those of $K(\omega)$ are given in Figure 3. Note that the horizontal axis denotes the real frequency, not the dimensionless frequency. Although only a second-order filter is used the match is very good. A more complex impedance function would have required a higher-order filter. With the filter identified, we can calculate the soil reaction force, $F(t)$, for the foundation as

$$F(t) = -1.1599F(t-1) - 1.1599F(t-2) + K_{static} \cdot \left[4.3033u(t) - 1.9780u(t-1)\right] \tag{10}$$

Such an expression can easily be incorporated in any structural time-history analysis program to account for SSI effects. Examples of such applications can be found in Safak (2004).

CONCLUSIONS

The frequency-dependency of foundation impedance functions makes it difficult to incorporate soil-structure interaction effects in standard time-history analysis software for structures under seismic loads. This difficulty can be eliminated by representing impedance functions as a ratio of two complex polynomials with unknown coefficients. The coefficients are determined by the least-squares approximation to the target impedance function. The ratio of two complex polynomials is equivalent, in the time-domain, to a discrete-time recursive filter, which is a simple finite-difference equation representing the relationship between foundation forces and displacements. Such a conversion of impedance functions makes it very easy to incorporate soil-structure interaction in standard time-history analysis.

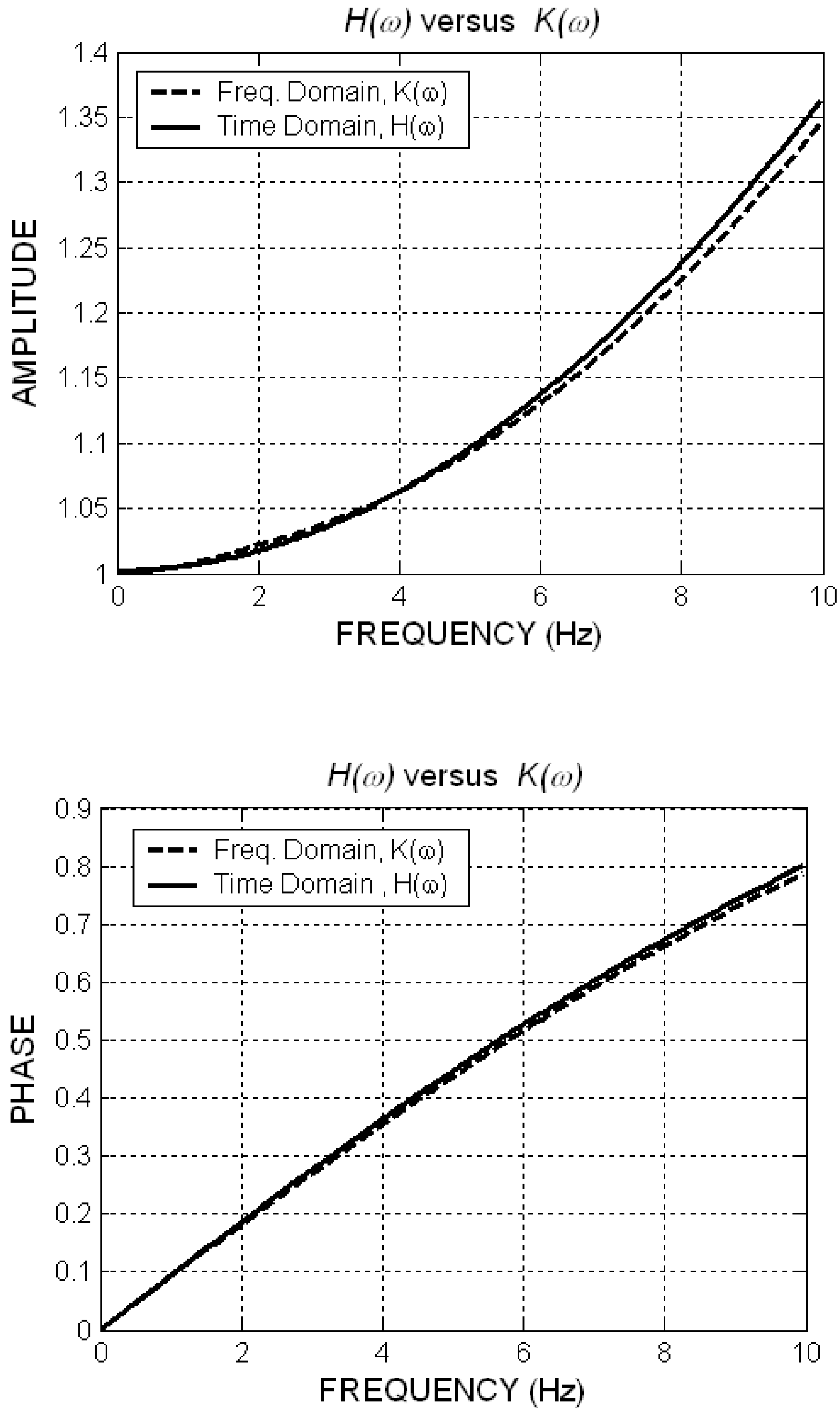

Figure 3 – Comparison of amplitude and phase of the foundation impedance function with those of the identified discrete-time filter.

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